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THEORETICAL RESEARCH ON LASER MODELS

Charles R. Willis

Boston University
Department of Physics
Boston, Massachusetts 02215

Contract No. AF 19(628)2460

Project No. 4645

Task No. 454502

FINAL REPORT

28 February 1966

Period Covered: 1 February 1963 to 31 March 1966

Prepared

for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
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ABSTRACT

Under the present contract we have developed a theory for the description and understanding of laser behavior from a fundamental microscopic point of view. Our model consists of N two-level systems interacting with a radiation field. We have treated problems of collective motion, coherence, dissipation and inhomogeneous broadening. This report consists mainly of the abstracts of six papers dealing with these subjects that have or will appear in scientific journals.

THEORY OF LASER MODELS

Introduction

The work performed under this contract consists in developing the consequences of a laser model of N two-level systems in interaction with a radiation field. The first paper in the series "A Model of Interacting Radiation and Matter" introduces the Hamiltonian of our model and the density matrix approach to the problem. In the first paper we treated atoms in the long wavelength limit and proved the self-consistent-field approximation is an exact solution in a well-defined limit. In the second paper Mr. Picard and the author of this report developed the coherence properties of our model. In the third paper we consider the equations of motion of a gas laser in the presence of dissipation. Paper four consists of an extension of the single mode case solved in paper three to the multimode gas laser. In paper five we consider previously neglected pump induced center of mass correlations. Paper six extends the results of paper four to the case where the average field vanishes. The last paper is a quantum mechanical solution of the density matrix equation for an inhomogeneously broadened gas laser to lowest order in the dimensionless coupling constant. The author is currently preparing a manuscript which includes a fully quantum mechanical treatment of the density matrix for a gas laser to fourth order in the dimensionless coupling constant.

Part I of this final report consists of seven abstracts of scientific articles, six of which have been published or are in press, and a seventh paper (paper number 6) which has appeared so far only

a scientific report.

Part II consists of three abstracts presented at American Physical Society Meetings. Two of the abstracts represent collaborations with graduate students, Mr. R. Picard and Mr. S. T. Scott, who have worked for short periods on the present contract.

Part I – Published Articles

1. "A Model of Interacting Radiation and Matter"

Charles R. Willis, J. Math. Phys. 5, 1241 (1964)

ABSTRACT

We investigate the long-time behavior of a model consisting of N two-level atoms in a lossless cavity. The Hamiltonian of our system contains the radiation oscillators in addition to the matter Hamiltonian and the usual $\int \underset{\sim}{j} \cdot \underset{\sim}{A} dv$ interaction term. In order to treat the system perturbatively, it would be neccssary to remove the tremendous degeneracy of the system. Since this is prohibitively difficult, and since we are interested in the long-time behavior of the system, we solve the quantum mechanical Liouville equation directly for a wide class of physically important initial distribution functions. We show the effective expansion parameter is $\tilde{\gamma}N\tilde{\gamma}$ where $\tilde{\gamma}$ is a dimensionless atomic dipole moment and N is the number of atoms. In the lowest order we find the self-consistent-field approximation. In the next order, particle-field correlations appear. We explicitly solve the equations of motion for the particle-field correlations in terms of the average quantities that appear in the self-consistent-field approximation. We show the self-consistent field approximation consists of five first-order differential equations.

Next we show the equations of motion for the density matrix of the system correct to order $(\tilde{\gamma}N\tilde{\gamma})^2$ are equivalent to eight first-order differential equations. The three additional equations are needed to describe the three second moments of the density matrix of the electromagnetic field that appear in second order. Our lowest-order microscopic equations are equivalent to semiphenomenological theories and our higher-order equations contain only the measurable second-order moments of the electromagnetic field in addition to the variables that appear in semiphenomenological theories.

2. "Coherence in a Model of Interacting Radiation and Matter"

Richard H. Picard and Charles R. Willis

Phys. Rev. 139, A10 (1965)

ABSTRACT

Some preliminary results of a theoretical investigation of the origin of laser coherence properties are presented. A model consisting of N two-level atoms in a perfect cavity interacting through their dipole moments with a single mode of the radiation field is adopted, and the resulting many-body problem is made solvable through the introduction of the self-consistent-field approximation (SCFA). It is shown that the SCFA is consistent with Glauber's suggestion that an ideal laser might be in a "coherent state", or pure eigenstate of the positive-frequency part of the electric-field operator. A conservation law is derived which assures the positive definiteness of the field-density operator, and it is shown that Glauber's coherent state is that solution of the SCFA equations

which minimizes the corresponding constant of the motion. When the SCFA is modified by the introduction of particle-field correlations, the conservation law is destroyed, and the coherent state is no longer an allowed solution.

3. "A Model of Interacting Radiation and Matter. II"

Charles R. Willis

Published in J. Math. Phys. 6, 1984 (1965)

ABSTRACT

We extend the investigation of the long time behavior of a model consisting of N two-level atoms interacting with radiation to include dissipation, pumping, and center of mass motion. We show that when the effective expansion parameter $\tilde{\gamma}N\tilde{\gamma}$ is small, the self-consistent-field approximation remains the solution to lowest order in $\tilde{\gamma}N\tilde{\gamma}$. The inclusion of the center of mass motion introduces another dimensionless parameter β into the theory. We show that when this parameter is small the electromagnetic field amplitude varies slowly on the time scale of the center of mass motion. We solve the equations of motion in this slowly varying limit by an extension of the Bogoliuboff-Kryloff theory of quasi-linearity to the problem of time dependent integral kernels. We find the unique stable stationary state and show that in the slowly varying limit the stationary state is approached independent of initial conditions. We calculate the frequency shift to second order in β. The first order frequency shift is the same as that calculated by Lamb. We compare our steady state solution with recent experiments with lasers. We include the effect of collisions in the steady state.

4. "Model of Interacting Radiation and Matter. III. Multimode Gas Lasers"

Charles R. Willis

J. Math. Phys., in press

ABSTRACT

We e[illegible] investigation of the long time behavior of a model consisting of N two-level atoms interacting with a single electromagnetic cavity mode to include interaction with many cavity modes. We show that, as a consequence of the coupling between radiation modes produced by spatial density variations of the population inversion, there is no strictly stationary state possible for multimode behavior. However, we obtain a stationary state by neglecting the rapidly oscillating terms. The steady-state population inversion is then a solution of an eigenvalue problem. The eigenvalue determinant is a function of the number of modes and the coupling between modes in addition to the usual dependence on frequencies and relaxation times. We explicitly solve for the unique eigenvalue in special cases. The corresponding eigenvector gives the steady-state mode intensity ratios. The absolute values of the steady-state intensities are determined by the energy conservation equation generalized to include pumping and dissipation. We also calculate the steady-state frequency shifts for each mode. The mode frequency shifts are practically independent of each other and have the same functional form as the single-mode frequency shifts.

5. "Models of Interacting Radiation and Matter for Gas Lasers"

Charles R. Willis

Proceedings of the Physics of Quantum Electronics Conference,

to be published

ABSTRACT

We investigate a model of N two-level atoms interacting with radiation in the presence of dissipation and pumping. First we show that the self-consistent-field approximation is a solution of our equations to lowest order in the dimensionless coupling constant α. The definition of α is $(\eta r_0 \lambda^2)^{1/2}$ where η is the number of two-level atoms per unit volume, r_0 is the classical electron radius, and λ is the wavelength of the radiation. When we consider the center of mass motion we get a second dimensionless constant, $\beta = \alpha(\omega_0/\omega_D)$ where ω_0 is the atomic frequency and ω_D is the Doppler width. When β is small the electromagnetic field amplitude varies slowly on the time scale of the center of mass motion. We find a unique stable stationary state and show that it is approached rapidly, independent of initial conditions. We find solutions of our equations corresponding to zero average field with the same steady state energy density and cavity frequency shift as in the self-consistent-field approximation. When pump induced correlations are present, we also show using perturbation theory that the solutions of our equations corresponding to zero average field have the same steady state energy density and cavity frequency shift as the average field theories of Lamb and Haken and Sauermann.

7. 8.

6. "Models of Gas Lasers"

Charles R. Willis

To be published

ABSTRACT

We extend the investigation of a model of N two-level atoms interacting with radiation in the presence of dissipation and pumping to include physical situations where the average fields vanish and where pump induced correlations are important. The dimensionless dynamical constant which determines the nature of the solution is $\beta^2 \equiv (\omega_L/\omega_D)^2$ where ω_L is $(4\pi)^{-1/2}(\eta r_0 \lambda^2)^{1/2}\omega_0$. η is the number of two-level atoms per unit volume, r_0 is the classical electron radius, λ is the wavelength of the radiation and $\hbar\omega_0$ is the two-level energy difference. The Doppler frequency ω_D characterizes the center of mass motion. In gas lasers $\beta \ll 1$ and, consequently, the nonlinear differential integral equations which determine the average value of the electromagnetic fields reduce to soluble nonlinear differential equations. We show the reduction of the equations of motion apply to the operator equations themselves and not just to the average values. Consequently, we are able to show that the steady state properties to a given order in β are the same whether the average field is zero or nonzero. When the pump induces correlations between the internal variables and the center of mass variables of the atom, we must use perturbation theory and our results when the average field is nonzero are equivalent to the theories of Lamb and Haken and Sauermann. We show that the steady state properties with pump induced correlations and average field zero are equivalent to the results

when the average field does not vanish. Consequently, steady state phenomena such as power dips, mode pulling, etc. are present when the average field vanishes and have the same functional dependence as the nonvanishing average field results.

7. "Quantum Theory of a Laser Model"

Charles R. Willis

to be published

ABSTRACT

We derive the kinetic equations for the coupled single particle density matrix ρ and the electromagnetic density matrix R to lowest order in the dimensionless coupling constant β^2 where $\beta^2 \equiv (\omega_L/\omega_D)^2$. The laser frequency ω_L is $(4\pi)^{-1/2}(\eta r_0\lambda^2)^{1/2}\omega_0$, where η is the number of two-level systems per unit volume, r_0 is the classical electron radius, λ is the wavelength of the radiation, and $\hbar\omega_0$ is the two-level energy difference. The Doppler frequency ω_D characterizes the center of mass motion. For gas lasers β^2 is much less than one and, consequently, we generalize and use the Bogoliubov derivation of kinetic equations for weak interactions. We find solutions when the average field vanishes and which include spontaneous emission correctly. The single particle density matrix and the radiation density matrix are coupled through their second moments. When we substitute the solution of the second moment equations into the density matrix equations, we find that each density matrix satisfies an uncoupled linear equation with known time dependent coefficients. We introduce and discuss dissipation from the density matrix point of view. With the use of the density matrix formalism we indicate that the correct expansion parameter for higher order kinetic equations is β^2.

Part II — Papers Presented at American Physical Society Meetings

1. "Model of Interacting Radiation and Matter"

Charles R. Willis

Bull. Am. Phys. Soc. 9, 399 (1964)

ABSTRACT

We have solved for the long-time behavior of a model consisting of N 2-level atoms in a lossless cavity. The Hamiltonian of our system contains the radiation oscillators in addition to the matter Hamiltonian and the usual $\int \underline{j} \cdot \underline{A} dv$ interaction term. In order to treat the system perturbatively, it would be necessary to remove the tremendous degeneracy of the system. Since this is prohibitively difficult and since we are interested in the long-time behavior of the system, we have solved the quantum-mechanical Liouville equation directly for a wide class of physically important initial-distribution functions. We show the effective expansion parameter is $\tilde{\gamma}N\tilde{\gamma}$, where $\tilde{\gamma}$ is a dimensionless atomic-dipole moment and N is the number of atoms. In the lowest order, we find the self-consistent-field approximation. In the next order, particle-field correlations appear. We have explicitly solved the equations of motion for the particle-field correlations in terms of the average quantities that appear in the self-consistent-field equations.

2. "Coherence in a Model of Interacting Radiation and Matter"

Richard H. Picard and Charles R. Willis

Bull. Am. Phys. Soc. 10, 98 (1965)

Some preliminary results of a theoretical investigation of laser coherence properties are presented. A model consisting of N 2-level

systems in a perfect cavity interacting through their dipole moments with a single mode of the radiation field is adopted, and the resulting many-body problem is made solvable through introduction of the self-consistent-field approximation (SCFA).[1] It is shown that the SCFA is consistent with Glauber's assumption[2] that an ideal laser is in a "coherent state", or pure eigenstate of the positive frequency part of the electric-field operator. A conservation law is derived that assures the positive definiteness of the field density operator, and it is shown that Glauber's state is that solution of the SCFA equations that minimizes the corresponding constant of the motion. When the SCFA is modified by the introduction of particle-field correlations, the conservation law is destroyed, and the coherent state is no longer an allowed solution. Various extensions of the results are considered briefly.

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[1] C. R. Willis, Bull. Am. Phys. Soc. 9, 399 (1964); J. Math. Phys. 5, 1241 (1964).
[2] R. J. Glauber, Phys. Rev. 130, 2529 (1963).

3. "Dynamical Coherence Time in a Gas Laser"

Samuel T. Scott and Charles R. Willis

Bull. Am. Phys. Soc. 10, 437 (1965).

We use an extended self-consistancy argument to study the dynamical dephasing of the field of a gas laser operating in a stationary state. For this purpose, the dependence of the atomic-inversion operators on the atomic velocities is crucial. The model includes dissipation, pumping, and external radiation noise. We consider the atomic position operators

to be time-dependent c numbers and make the rotating-wave approximation so that the interaction Hamiltonian contains only terms on the "energy shell". We derive the equations of motion for the atomic-inversion operators and for field-correlation operators $a^{+}(t)a(t-\tau)$ and average over a factorized density matrix. We consider the stationary limit where $\langle a^{+}(t)a(t-\tau)\rangle = C(\tau)$ and solve for $C(\tau)$ self-consistently in terms of the external pump and the noise power. $C(\tau)$ determines the power density spectrum and a complex frequency shift. The imaginary part of the frequency shift is just the usual frequency shift, and the real part is the reciprocal coherence time as measured in a Michelson interferometer. The noise is shown to contribute negligibly to the frequency shift and coherence time of a gas laser above threshold, but causes a small increase in the pumping threshold.

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